Cognitive Radio Networks: Optimization of Cooperative Spectrum sensing to minimize the total error rate

Gaurav G. Bhosale 1, Dipak B. Khandgaonkar 2 and J. Christopher Clement 3
Student, School of Electronics Engineering, VIT University, Vellore - 632014, TamilNadu, India 1
Student, School of Electronics Engineering, VIT University, Vellore - 632014, TamilNadu, India 2
Professor, School of Electronics Engineering, VIT University, Vellore - 632014, TamilNadu, India 3
bhosale_gaurav@yahoo.co.in1, khandgaonkar.dipak@gmail.com2, christopher.clement@vit.ac.in3

Abstract-In this paper, by using cooperative spectrum sensing and adapting to the environment, a cognitive radio is able to fill spectrum holes and serve without causing harmful interference to the licensed user. We consider optimization of cooperative spectrum sensing with energy detection to minimize the total error rate. We derive optimal voting rule for optimal value of cognitive radios out of many. We overcome the hidden terminal problem by using multiple cognitive radios. We also determine optimal detection threshold to minimize error rate. For large number of cognitive radios, we propose efficient sensing algorithm for a given error bound to optimize number of Cognitive radios.

Keywords- Cognitive radio, Optimization, Energy detection, Fusion centre, Spectrum sensing.

I. INTRODUCTION

In the wireless communication, spectrums are the valuable resources. In wireless service, particular frequency band is allotted to specific user only so that other user can not access others information. Due to popularity of wireless communication needs more frequency band and this reflects into spectrum scarcity problem. Most of the radio frequency spectrums are inefficiently utilized. In most parts of the world, cellular telephone bands are overcrowded. The other frequency bands like television, military, amateur, radar, etc are partially utilized. Survey of Federal Communication Commission (FCC) concluded that spectrum utilization depends on time and place [5]. To reduce spectrum scarcity and conflict problem, Cognitive Radio (CR) technology is introduced. It allows secondary user to borrow unused spectrum of primary user. Cognitive radio is a system, which knows the radio frequency environment, it sets parameters like frequency, bandwidth, power and senses the spectrum hole in the primary user band and assign that spectrum to secondary user.

The main problem in sensing the spectrum is hidden terminal problem [6]. That is, the primary user (PU) is out of range of a CR then there is chance of false assigning of spectrum. This problem is solved by using number of cognitive radios and sensing the spectrum simultaneously. The spectrum sensing is carried by large number CR’s. This increases processing time of decisions carried out by CR’s and in mean time there is possibility of wrong decision. This problem is overcome by using voting rule and optimization of CR’s. In this paper, we consider optimization of co-operative spectrum sensing with energy detection to minimize the total error rate. Each CR performs energy detection of the signal received [2-4]. Then it takes decision and sends it to fusion centre. Using “n out of k rule”, we optimize the number CR’s[1]. We also optimized the detection threshold for minimum error rate. For large number of CR’s, we determined fast sensing algorithm, which shows that with help of few CR’s we can achieve target error bound[1].

The paper contains, cooperative spectrum sensing, the optimization of cooperative spectrum sensing, the optimal voting rule, the optimal threshold and fast spectrum sensing method. We have discussed results and concluded at the end of paper.

II. SYSTEM MODELLING

A] Spectrum Sensing

We consider cognitive network with K number of CR’s, one primary user and one fusion centre (i.e. common receiver). The spectrum sensing is done by each CR independently. The decisions taken by CR are sent to the fusion centre then fusion centre will decide that primary user is present or absent. We consider two hypotheses [1];
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H₀: The primary user is absent.

H₁: The primary user is in operation.

When the signal is received at each $i^{th}$ CR, it follows two hypotheses as above. Then the received signal will be:

$$x_i(t) = \begin{cases} w_i(t), & H_0 \\ h_i(t) s(t) + w_i(t), & H_1 \end{cases} \quad \text{.........}(1)$$

where, $x_i(t)$ is the received signal at the $i^{th}$ CR in time slot $t$, $s(t)$ is the PU signal. The $h_i(t)$ indicates the complex channel gain of the sensing channel between the PU and $i^{th}$ CR. $w_i(t)$ is the AWGN (Additive White Gaussian Noise). We assume the sensing time is lesser than coherence time of the channel. The coherence time is the time duration over which the channel impulse response is considered to be not varying. Therefore $h_i(t)$ will be time-invariant so $h_i(t)$ is equal to $h_i$ (i.e. time independent). Also we assume that during sensing time, PU does not change its state. We go for energy detection technique as PU signal is unknown. For each $i^{th}$ CR by energy detection we find average probability of detection, missed detection and false alarm over AWGN channel with following equations [1];

$$p_{f,i} = \frac{\Gamma\left(\frac{u, \lambda_i}{2}\right)}{\Gamma(u)} \quad \text{.........}(2)$$

$$p_{d,i} = Q_u\left(\sqrt{2\gamma_i}, \sqrt{\lambda_i}\right) \quad \text{.........}(3)$$

$$p_{m,i} = 1 - P_{d,i} \quad \text{.........}(4)$$

Where, $\lambda_i$ is the energy detection threshold and $\gamma_i$ is the instantaneous signal to noise ratio (SNR) at the $i^{th}$ CR. Also $u$ is the time-bandwidth product of the energy detector, $\Gamma(a)$ is the gamma function and $\Gamma(a,x)$ is the incomplete gamma function is equal to [1].
The generalised Marcum Q-function i.e. \( Q_a(b) \) is given by [1,7]:

\[
Q_a(a,x) = \frac{1}{a^{\frac{x}{2}}} \int_x^{\infty} t^{\frac{a+1}{2}} e^{-\frac{t^{2}+a^2}{2}} I_{a-1}(at) dt 
\]

Where \( I_{u-1}(.) \) is the first kind and order \( u-1 \) modified Bessel function.

The cooperative spectrum sensing, where number of CR’s takes binary decision based on local observation and forwards a bit decision \( D_i \) to the common receiver. These decisions are summed at common receiver and it will decide whether the PU is absent or in operation [1].

\[
Y = \sum_{i=1}^{K} D_i \begin{cases} 
\geq n, \ H_1 \\
< n, \ H_0 
\end{cases}
\]

Here, \( n \) is the threshold representing “n-out-of-K” rule. If the number of CR is one, i.e. \( n=1 \) then it corresponds to OR rule and if \( n = K \) then it corresponds to AND rule.

In the radio frequency environment around CR’s, we consider the distance between any two cognitive radios is smaller than the distance between one CR and PU. Therefore the signal received at each CR follows identical path loss. For AWGN channel, \( \gamma_1 = \gamma_2 = \ldots = \gamma = \gamma_k \) and for Rayleigh fading channel \( \gamma_1, \gamma_2, \ldots, \gamma_k \) as we assume that it is independent and identically distributed (i.i.d) with instantaneous SNR’s. Also, these SNR’s are i.i.d. exponentially distributed random variables with the same mean. We consider another assumption that threshold of each CR is same and it is \( \lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda \). As threshold is constant for all CR, \( P_{f,i} \) will be independent of \( i \), therefore \( P_{f,i} = P_{f} \). For AWGN channel, \( P_{d,i} \) is independent of \( i \) and we denoted as \( P_{d} \). In Rayleigh fading channel, \( P_{d} \) is \( P_{d,i} \) averaged over the different values of \( \gamma_i \) [1-3].

The common receiver calculates false alarm probability and missed detection probability with the help of average probability of each CR. The false alarm probability is given by [1],

\[
Q_f = \sum_{l=n}^{k} \binom{k}{l} p_f^l (1-p_f)^{k-l} = \text{Prob} \{ H_1 / H_0 \} 
\]

Also, the missed detection probability is given by,

\[
Q_m = 1 - \sum_{l=n}^{k} \binom{k}{l} p_d^l (1-p_d)^{k-l} = \text{Prob} \{ H_0 / H_1 \} 
\]
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B) Optimization of Cooperative Spectrum Sensing

In this section, we analyse optimal voting rule, optimization of number of CR and detection threshold with cooperative spectrum sensing.

1. Optimal Voting Rule

Let, K is fixed then what is will be optimal value of n so that we get minimum error rate \(Q_f + Q_m\), this is the optimal voting rule and optimal value of n is called as \(n_{opt}\). We have plotted graph for n=1 to n=10. For each n, for different threshold values, we calculated error rate. For small threshold value, we get more error rate and optimal rule AND rule (i.e. \(n = 10\)). For large threshold value, optimal rule is OR rule. But when \(n = 5\), we get more error rate for medium threshold values [1].

Statement 1: To find \(n_{opt}\) value for minimum error rate we proposed solution as follows:

\[
\begin{align*}
    n_{opt} &= \min \left( K, \left[ \frac{K}{1 + \alpha} \right] \right) \\
    \ln \frac{P_f}{1 - P_m} &= \frac{\ln \frac{P}{1 - P_m}}{\ln \frac{P}{1 - P_f}} \\
    \text{where, } \alpha &= \frac{P}{1 - P_m} \text{ and } [.] \text{ denotes the ceiling function}
\end{align*}
\]

\[\ldots \ldots \ldots \ldots (9)\]

Proof:

From equation 7 and 8, we get, \((Q_f + Q_m) = 1 + G(n)\). For optimal value of \(n\), error rate should be minimum,[1] i.e.

\[
\frac{\partial G(n)}{\partial n} = G(n + 1) - G(n)
\]

Therefore, the difference is given by \(G(n + 1) - G(n)\).

\[
\frac{\partial G(n)}{\partial n} = G(n + 1) - G(n)
\]
\[
= \sum_{l=n+1}^{K} \left(\begin{array}{c} K \\ l \end{array}\right) \left[ P_f^l(1-P_f)^{k-l} - (1-P_m)^l P_m^{K-l} \right] - \sum_{l=n}^{K} \left(\begin{array}{c} K \\ l \end{array}\right) \left[ P_f^l(1-P_f)^{k-l} - (1-P_m)^l P_m^{K-l} \right]
\]
\[
= \left(\begin{array}{c} K \\ n \end{array}\right) \left[ -P_f^n(1-P_f)^{k-n} + (1-P_m)^n P_m^{K-n} \right]
\]
i.e. only \( l = n \) term remains
\[
\therefore -P_f^n(1-P_f)^{k-n} + (1-P_m)^n P_m^{K-n} = 0
\]
\[
\therefore P_f^n(1-P_f)^{k-n} = (1-P_m)^n P_m^{K-n}
\]
On simplifying, we get

\[
n \approx \left[ \frac{K}{1 + \alpha} \right] \quad \text{where} \quad \alpha = \frac{\ln P_f}{1 - P_m} \cdot \frac{\ln P_m}{1 - P_f}
\]

From equation 9, we get some values for \( n \):

\( a) \) If \( P_f \) and \( P_m \) are of same order then \( \alpha = 1 \) and \( n=K/2 \)
\( b) \) If \( P_f \leq P_m^{k-1} \) results in \( P_f<<P_m \), for large \( K \) then \( \alpha \geq K-1 \) and \( n=1 \), i.e. OR rule.
\( c) \) If \( P_m<<P_f \) then \( \alpha \) tends to zero and \( n=K \), i.e. AND rule.

2. Optimal Energy Detection Threshold

Here we consider that \( K, n \) and SNR are known then what will be optimum threshold \( \lambda^* \) such that total error rate minimum. We have plotted in figure 1 total error rate curve with different threshold values. For only one threshold value, figure has the low error rate for given \( n \). i.e. there will be one and only value of \( \lambda \) for which \( (Q_f + Q_m) \) is minimum [1]

\[
\lambda^* = \arg\{\min(Q_f + Q_m)\}
\]  \[\text{..................(10)}\]

For optimal energy threshold;

\[
\frac{\partial Q_f}{\partial \lambda} + \frac{\partial Q_m}{\partial \lambda} = 0
\]
\[
\frac{\partial Q_f}{\partial \lambda} = \sum_{l=m}^{K} \left(\begin{array}{c} K \\ l \end{array}\right) \left[ lP_f^{l-1}\frac{\partial P_f}{\partial \lambda}(1-P_f)^{k-l} - \sum_{l=m}^{K} \left(\begin{array}{c} K \\ l \end{array}\right) P_f^l(K-l)\frac{\partial P_f}{\partial \lambda}(1-P_f)^{K-l-1} \right]
\]
\[
\frac{\partial Q_m}{\partial \lambda} = \sum_{l=n}^{K} \left(\begin{array}{c} K \\ l \end{array}\right) lP_d^{l-1}\frac{\partial P_d}{\partial \lambda}(1-P_d)^{k-l} + \sum_{l=n}^{K} \left(\begin{array}{c} K \\ l \end{array}\right) P_d^l(K-l)\frac{\partial P_d}{\partial \lambda}(1-P_d)^{K-l-1}
\]

3. Optimal Number of Cognitive Radios

In cooperative spectrum sensing, large number of CR’s used, but it increases the time slot and becomes impractical. As only one CR should send its local decision at a time to the fusion centre so it may take whole sensing time intolerably long. This
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problem can be solved by permitting CR’s to send the decision concurrently but this is difficult for common receiver to separate each decision. There is another way to send decisions using orthogonal frequency bands, but large bandwidth requirement is the problem. So we proposed efficient sensing algorithm, in which we define some error bound and calculated optimal number of CR’s. Also each CR sends decision in one time slot. By this method we get required error rate with use of few CR’s only.

Let, SNR and threshold values are known then we calculated least number CR’s in cooperative spectrum sensing to achieve target error bound. i.e. \( (Q_f + Q_m) \leq \varepsilon \), where \( \varepsilon \) is the target error bound. As we have stated earlier for optimal voting rule [1],

\[
n^\text{opt}_K = \min \left( K^*, \left[ \frac{K^*}{1 + \alpha} \right] \right)
\]

Here, \( K^* (1 \leq K^* \leq K) \) is the least number of CR’s to satisfy target error bound \( (Q_f + Q_m) \leq \varepsilon \) and \( \alpha \) is calculated from \( P_f, P_m \) and known SNR and \( \lambda \) values. We define the function,

\[
F(k, n^\text{opt}_k) = Q_f + Q_m - \varepsilon
\]

where \( k \) is the number of cooperative CR’s in cooperative spectrum sensing and \( n^\text{opt}_k \) is calculated from equation 9. The probability \( Q_f \) and \( Q_m \) are functions of \( k \) and \( n^\text{opt}_k \). Therefore we get;

\[
F[k^*, n^\text{opt}_{k^*}] \leq 0
\]

\[
F(k^* - 1, n^\text{opt}_{k^* - 1}) > 0
\]

Using above equations we can get \( k^* = \lfloor k_0 \rfloor \), where \( k_0 \) is the first zero crossing point of the function \( F(k, n^\text{opt}_k) \) in terms of \( k \). Therefore, fast sensing algorithm can be implemented by considering only \( k^* \) CR’s instead of \( K \). This reduces the time slot for common receiver from \( K \) to \( k^* \) maintaining the target error bound.

III. SYSTEM MODELLING WITH ENERGY DETECTION OF SIGNAL

Here, the energy of signal is calculated and probability of false alarm and detection is calculated.[2-4] For AWGN channel, first we define different threshold values and calculate the energy of received signal. If energy of received signal is \( x_i(t) = s(t) + w(t) \), then the energy of \( x_i(t) \) is calculated, also if received signal is \( x_2(t) = w(t) \), then energy of \( x_2(t) \) is calculated. If energy of \( x_1(t) \) is greater than threshold value then that would be probability of detection and if energy of \( x_2(t) \) is greater than threshold value then that would be probability of false alarm. [2]

\[
E_i = \frac{1}{N_{02}} \sum_n (s(n) + w(n))^2 \quad \text{And} \quad E_2 = \frac{1}{N_{02}} \sum_n (w(n))^2
\]

where \( N_{02} \) is the two sided noise power spectral density [2] and is given by;

\[
N_{02} = \frac{\sum_n (s(n))^2}{(2 \times \text{SNR})}
\]
For Rayleigh Fading Channel, the SNR values are exponentially distributed. We consider SNR values as exponential random number with same mean. To determine Rayleigh fading channel gain we have used [3]

\[ h = \sqrt{\frac{(2 \times \text{SNR})}{\sum_n (s(n))^2}} \]

(15)

Then we find the two sided noise power by [3]

\[ N_{02} = \frac{\{h^2 \times \sum_n (s(n))^2\}}{(2 \times \text{SNR})} \]

(16)

Then using this value of \(N_{02}\) and equation (11) we calculated the energy of the received signal and find probability of false alarm and detection using threshold values.

The energy becomes in Rayleigh Fading Channel;

\[ E_1 = \frac{1}{N_{02}} \sum_n \left( h \times s(n) + w(n) \right)^2 \]

IV. RESULTS AND DISCUSSIONS

In the figure.1, we found error rate for different threshold values and number of CR’s by keeping SNR=10 db. In figure, the error rate is low for \(n = 5\) and it is high \(n = 10\) and \(n = 1\), i.e. with use of 5 CR’s out of 10 we can achieve low error rate. This figure explains the optimal rule. The error rate is nothing but \((Q_f + Q_m)\). That is probability of missed detection and false alarm probability is high if very few or high number CR’s are used. So the number of CR’s used should be half of total CR’s, i.e. for \(n = 5\) the probability of missed detection and false alarm probability is low, so cooperative spectrum sensing allocation is done in correct way.

Also, by modelling the system, we compare results get from modelling and formulae for \(n = 5\). The both results are same. For modelling, we use equations explained in section 4.
Fig. 1. Total error rate of cooperative spectrum sensing in AWGN channel with 10dB SNR. Optimal voting rule for n=1,2,........,10 and K=10.

In the figure 2, we found optimum value of ‘n’ i.e. ‘n out of K’ CR’s. We vary threshold values from 10 to 40 and for different SNR values (0dB, 5dB, 10dB), we found optimal value of ‘n’ from equation 9. From graph we conclude that for low threshold value with low SNR, the required number of CR’s is more. As we increase threshold value with low or same SNR then we requires very less number of CR. Also as SN increases, the optimal value of n increases. E.g. If SNR= 0dB and threshold = 33 then optimal value of n is 1. That is with 1 CR we can achieve low error rate.

For high threshold value, optimal value of n is small, so for high threshold value with less number of CR’s, we get low probability of missed detection and false alarm probability. Also, this probability reduces by decreasing SNR values for small number of ‘n’ in AWGN channel.
In the figure 3, take an example for fig.3, here we consider 50 CR’s in AWGN channel with SNR=10 dB. We applied optimal voting rule for threshold values 30, 40 and 50. For $\lambda=30$, we get error rate $10^{-2}$ with use of 8 CR only. Also, if we increase threshold value then the least number of CR’s increases for target error bound. e.g. For $\lambda = 50$, to get error rate 0.01, we need 30 CR’s out of 50, so as threshold value increases number CR’s require for target error bound also increases. If we want more error bound like 0.0001 then least number required CR’s increases.

This figure shows that with least number of CR’s, we can achieve less probability of missed detection and false alarm probability (i.e. error rate). For low threshold value, with less number of CR’s we achieve low probability of missed detection and false alarm. This increases probability of detection and fusion centre can correctly allot spectrum to the secondary user with use of few CR’s only.

Also, use of few CR’s decreases processing time of common receiver as it checks decisions of few CR’s only. This decreases sensing time and increases probability of correct allocation of spectrum. This is the in short fast sensing algorithm.
In the figure, we find error rate in Rayleigh fading channel using modelling of system by energy detection technique. Here, the SNR values are exponentially distributed random variables with same mean. We find $n$ optimal values from equation 9. From optimum $n$, we find $Q_f$ and $Q_m$. The SNR varies from 5, 10, 15, 20 dB with threshold value $\lambda = 20$. The error rate 0.01 is achieved with use of 12 CR out of 50 for SNR= 20 dB. As we decreases SNR values, the required number CR’s for target error bound is increases. So for Rayleigh fading channel, the least number of CR is more for less SNR for a constant threshold value.

For Rayleigh Fading channel, the probability of missed detection and false alarm probability is high for low SNR values. For high SNR values, probability of correct detection achieved with use of less CR’s than low SNR values. Over Rayleigh fading channel, SNR values should be greater than or equal to 10dB to get least number of CR’s in cooperative spectrum sensing and to maintain target bound. (i.e. more detection probability and less probability of missed detection and false alarm.)
Fig.4. Total error rate for 50 CR’s in Rayleigh Fading channel with SNR values 5dB, 10dB, 15dB and 20dB with optimal voting rule and energy threshold value $\lambda=20$

V. CONCLUSION

We have studied the cooperative spectrum sensing with energy detection using formula and modelling the system. We analysed the system with optimum voting rule for minimum error rate and $K/2$ is optimal value. Also, optimization of threshold has been done with minimum values of probability of missed detection and false alarm probability. We analysed the system, for the less probability of missed detection and false alarm probability so that spectrum allotted correctly to secondary user. We proposed the fast sensing algorithm and calculated least number of CR’s for a given error bound. We eliminated the intolerably long sensing time with fast sensing algorithm.

REFERENCES


